ONTO-SEMIOTIC ANALYSIS OF A LESSON ON PERCENTAGES

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Abstract

Percentages are an important topic in the mathematics curriculum, both in primary education and in compulsory levels of secondary education. At the same time, the textbook is a widely used resource in the practice of teaching. We start from the basis that the teachers should have the required knowledge to evaluate how the topics included in the school texts are introduced and to decide consequently about their relevance and didactical suitability. Therefore, in this work we develop a procedure to analyze, from a didactical point of view, a lesson on percentages of a sixth-grade primary school textbook, using the Onto-Semiotic Approach to mathematical knowledge and instruction as theoretical-methodological framework. Then, the didactical suitability of the lesson is assessed considering specific suitability criteria inferred from previous research on the subject. The results reflect that the sample of tasks in which the percentages must be contextualized and applied is not sufficiently representative. Besides, possible conflicts of meaning and potential improvements that should be considered in the design of educational textbooks are shown.

Keywords: primary education, mathematics education, percentages, textbooks, onto-semiotic approach, didactical suitability.

1 INTRODUCTION

Percentages are a fundamental concept in daily life ([1]), present both in the curriculum of primary education and secondary education. However, this notion is not well understood, and people make mistakes when using and interpreting them, such as arithmetic errors or applying the rule of three to non-proportional situations. The students solve the problems and exercises that involve this concept simply putting into practice the algorithms they have learn, but with a poor conceptual knowledge, without taking into account their different representations and meanings ([2]).

An important aspect to consider is that a task can afford many opportunities for learning percentages, which are hidden. Thus, as Lundberg and Kilhamn ([3], pp. 576) point out, a mathematical task presented in a textbook can not be judged as good or bad in itself: “the learning opportunities a task designer sees in a task may not stand out as to the teacher and consequently not included in the transposition into knowledge actually taught”. On the other hand, it cannot be ignored that the teacher's use of textbook lessons determines to a large extent the student's study process, being, furthermore, considered “a habitual resource in the development of the teaching and learning process, to the point that, on many occasions, it is the manual itself that determines the real curriculum” ([4], pp. 38). Consequently, teachers should have the necessary knowledge —subject matter knowledge as well as didactical content knowledge, to evaluate the characteristics of the topics included in school texts and decide on their relevance and didactical suitability.

The purpose of this paper is to show a technique of mathematical text analysis based on the application of some notions from the Onto-Semiotic Approach to mathematical knowledge and instruction (OSA) ([5]), which offers a pragmatic approach that is suitable to study the multiple meanings of a mathematical object. The analysis is oriented to identify elements that can be potentially conflicting, from the point of view of the intended mathematical knowledge and the proposed teaching-learning process. This information can be useful for the teacher who uses the book as a resource, in order to take into account such conflicts and to foresee possible ways to manage them.

This study is structured in the following way. Section 2 describes the research problem and theoretical framework. In section 3, we determine the types of institutional meanings associated to the percentage notion; we analyze the didactic configurations of the selected tasks; and we present the potential epistemic and cognitive conflicts derived from the technique of textbook analysis. Finally,

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section 4 is devoted to the discussion of the didactical suitability of the instructional process on percentages that is planned in this textbook. We conclude emphasizing the importance of carrying out this type of study and stand out some suggestions for teachers and textbook authors.

2 THEORETICAL FRAMEWORK AND METHODOLOGY

The theoretical framework we use in this research is the Onto-Semiotic Approach (OSA) of mathematical knowledge and instruction ([6], [5]). OSA framework offers some theoretical tools, such as those of pragmatic meaning, onto-semiotic configuration, didactical configuration, and didactical suitability, which allow a systematic didactical analysis of the instructional processes. Godino and Batanero consider mathematical practice to be “any performance or expression (verbal, graphic, etc.) made by someone to solve mathematical problems, communicate to others the solution obtained, validate or generalize it to different contexts and problems” ([6], pp. 334). These practices may be developed by a person or shared within an institution. Since a mathematical object, in its institutional version, is conceived as an “emerging system of social practices associated with a field of problems” ([6], pp. 335), the (institutional) meaning of an object is considered to be constituted by the “system of institutional practices associated with the field of problems from which the object emerges at a given moment” ([6], pp. 338).

To perform the analysis of mathematical practices, the OSA has introduced the notion of onto-semiotic configuration of practices, objects and processes, in which the various types of objects, according to their nature and function, are classified into the following categories:

- Languages (terms, expressions, notations, graphics) in their various registers (written, oral, gestural, etc.).
- Situations-problems (intra or extra-mathematical applications, exercises).
- Concepts-definition (introduced by definitions or descriptions).
- Propositions (statements about concepts).
- Procedures (algorithms, operations, calculation techniques).
- Arguments (statements used to validate or explain propositions and procedures, deductive or other kinds).

The onto-semiotic configurations can be of the cognitive type, when it is the personal mathematical objects that a subject mobilizes as part of the mathematical practice developed to solve a problem situation, or of an epistemic type, if such practice is considered from the institutional point of view. The emergence of the objects of the configuration takes place through the respective mathematical processes of: communication, problematization, definition, enunciation, elaboration of procedures (algorithmization) and argumentation.

Godino, Contreras, and Font [7] introduce the notion of didactic configuration as a tool for the analysis of mathematical instruction processes. A didactic configuration is a segment of teaching and learning activity that is distributed between the start and end moments of solving a task or situation-problem designed or implemented.

The didactic suitability is defined as the degree to which an instructional process (or a part of it) has certain characteristics that allow it to be described as optimal or adequate to achieve the adaptation between the personal meanings achieved by the students (learning) and the institutional meanings pretended (teaching), taking into account the circumstances and available resources (environment).

Considering the lesson of the textbook as an ‘instructional process’ (planned), in this work we focus on identifying the didactic-mathematical knowledge that can be extracted from the didactic analysis of the same, made by applying the theoretical tools described.

2.1 Meanings of percentage

Some researchers ([8], [9], [10]) have pointed out different meanings for percentages, involving the concepts of fraction, ratio and rational number and the dependence of each meaning with the context of use:

- Percentage, as a way to represent a number. The percent is here understood as a translation of the ‘%’ symbol. Thus, percentages can be transformed into real numbers that meet their axioms, can be ordered and added directly if they represent different parts of the same whole ([11]). In this change of representation, the change of the reference unit of the percentage from 100 to 1 should be noticed, which allows to use the expression 50% in its decimal form 0.5 or in its fractional form ½ when calculating.

- Percentage as intensive quantity. The percentage is understood as an internal ratio, that is, as a way to quantify multiplicative relationships between quantities of the same magnitude. For Parker and Leinhardt [12] when a quantification is expressed using percentages there is no doubt that the numeral is the expression of an intensive quantity linked to an internal ratio.
- Percentage as part-whole relation. In this context, the size of a subset is compared to the size of the set of which it is a part. The part-whole meaning is the standard in school textbooks, especially when it comes to introduce percentages. In this context, some obstacles arise, as percentages greater than 100 do not fit.
- Percentage as a part-part relation (ratio). The percentage is used as a representation of the part-part relationship, when it describes a comparison between different sets, different attributes of the same set, or to describe the change of a set over time.
- Percentage as a statistical index. The percentage informs or represents the relation of the relative size of a particular quantity in relation to another.
- Percentage as a function. The percentage is used as an operator that establishes a functional relationship between the initial quantity and the final quantity. It establishes a rate that allows determining quantities such as the final amount of the tax, discounts, interest, etc. Authors such as Davis [13] point to the functional use of percentage as its most important meaning.

Parker and Leinhardt [12] suggest that the meaning of percentage as real number has been lost in between the rules of change from decimals to fractions, fractions to decimals, improper fractions to mixed numbers and mixed numbers to improper fractions. The knowledge about percentages is much more than the successful calculation with them. More than this procedural mess of conversions, calculations and applications, comprehensive knowledge about percentages, both in the school and outside of it, should imply understanding their multiple and imbricated meanings and their relational character ([12], pp. 471).

Although the meanings of percentage as a mathematical object are diverse, the essence is the proportionality ratio. Thus, Parker and Leinhardt [12] consider that the percentage is fundamentally a privileged language of proportions that simplifies and condenses descriptions of multiplicative comparison (pp. 472). The percentage relates two quantities of directly proportional magnitudes, that is, if A and B are two directly proportional magnitudes, the percentage that the quantity A represents with respect to the quantity B is the quantity of A that corresponds to one hundred B units.

Proportionality and percentages are contemplated in different thematic blocks within the Spanish curriculum ([14]) for primary education. Table 1 shows the references to arithmetic proportionality and percentages in this national curriculum.

Table 1. Contents, evaluation criteria and evaluable learning standards.

Contents	Evaluation criteria	Evaluable learning standards
Percentages and Proportionality. Percentages: Expression of parts using percentages. Correspondence between simple fractions, decimals and percentages. Percentage increases and decreases. Direct proportionality. The rule of three in situations of direct proportionality: law of double, triple, half.	7. Be initiated in the use of percentages and direct proportionality to interpret and exchange information and solve problems in contexts of daily life.	7.4. The student uses the rule of three in situations of direct proportionality: law of double, triple, half, to solve problems of daily life. 7.5. The student solves problems of daily life using percentages and rule of three in situations of direct proportionality, explaining orally and in writing the meaning of the data, the situation, the process followed, and the solutions obtained.

3 ANALYSIS OF DIDACTIC CONFIGURATIONS. POTENTIAL EPISTEMIC AND COGNITIVE CONFLICTS

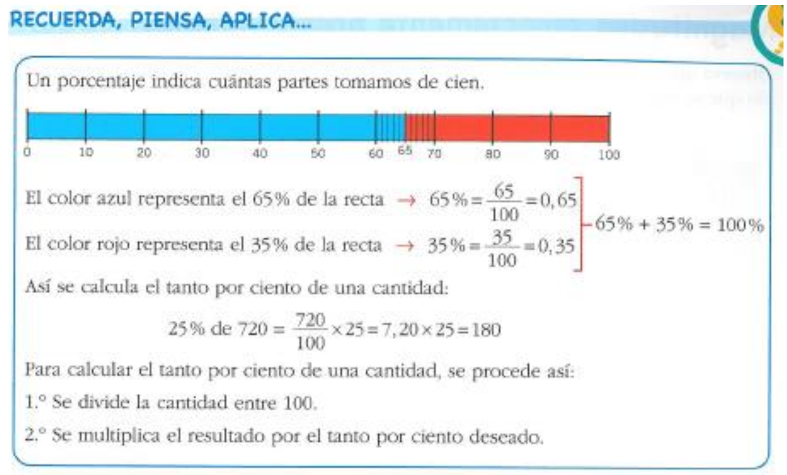
The text analyzed is part of unit 8 (Percentages and Proportionality) of the book by Ferrero, Martín, Alonso, and Bernal [15] for the sixth year of primary education and appears after the topic dedicated to the study of fractions. The lesson begins with a section in which a situation about using double, triple and half amounts of weight and value is described, in a context of preparation of a cooking recipe. Then, the concept of percentage is remembered as a written representation of rational numbers, and applied to find the percentage of a quantity. The concept of percentage and its application is considered again at the end of the lesson.

3.1 Didactical configurations

Next, we analyze the didactical configurations that the selected textbook devotes to percentages. To do this, we describe, in the first place, the operational and discursive mathematical practices that are proposed, and we identify the mathematical objects and processes that intervene in them. Next, we describe the main epistemic and cognitive conflicts that can be identified using our theoretical framework.

3.1.1 Didactical configuration 1. Remember, think, apply

This first configuration (Fig. 1) is intended to summarize what students should know about percentages.


RECUERDA, PIENSA, APLICA...

Un porcentaje indica cuántas partes tomamos de cien.

El color azul representa el 65% de la recta → $65\% = \frac{65}{100} = 0,65$

El color rojo representa el 35% de la recta → $35\% = \frac{35}{100} = 0,35$

65% + 35% = 100%

Así se calcula el tanto por ciento de una cantidad:

$$25\% \text{ de } 720 = \frac{720}{100} \times 25 = 7,20 \times 25 = 180$$

Para calcular el tanto por ciento de una cantidad, se procede así:

1.º Se divide la cantidad entre 100.

2.º Se multiplica el resultado por el tanto por ciento deseado.

REMEMBER, THINK, APPLY ...

A percentage indicates how many parts we take from one hundred.

The blue colour represents 65% of the line.

The red colour represents 35% of the line.

This is how the percent of a quantity is calculated:

To calculate the percentage of a quantity, we proceed as follows:
1º. The amount is divided by 100.
2º. The result is multiplied by the desired percent.

Figure 1. Configuration 1 (Remember).

Table 2 summarizes the objects that intervene in this configuration.

Table 2. Intervening objects in the didactical configuration 1.

Languages	Natural; symbolic (%, fraction, decimal number); graphic (length diagram)
Concepts	Percentage (percentage shows how many parts are taken out of one hundred) Multiplication, division, decimal fraction, decimal number Part-whole
Procedures	Calculating the percent of a quantity (the quantity is divided by 100 and then multiplied by the number that accompany it at the left side).
Propositions	When a rectangle is splitted in two, each of them being a percentage, their sum is equal to the whole, that is, the 100%. Particularized to the example: 65%+35%=100%.

The percentage is the (new) emerging concept that is intended to be introduced with this sequence of practices, while the concepts of multiplication, division, fraction, and decimal number are considered as previous, that is, previously acquired by the reader.

The most important conflict that appears here is that the situation-problem that gives meaning to the practices is not mentioned, leaving to the reader (or to the teacher who manages the implementation of the sequence) the responsibility to make sense for this speech. Implicitly, this situation-problem is evoked:

> How can one express the relative size of the parts marked in blue and red with respect to the total length?

Another conflict is the absence of argumentation or justification of the operative and discursive practices:

> The percentage is a way to write the fraction, understood as an expression of a part (numerator, 65) with respect to a whole (denominator, 100).

This configuration is a synthesis that summarizes the work that is going to be developed on percentages, or that is supposed to be previously studied.

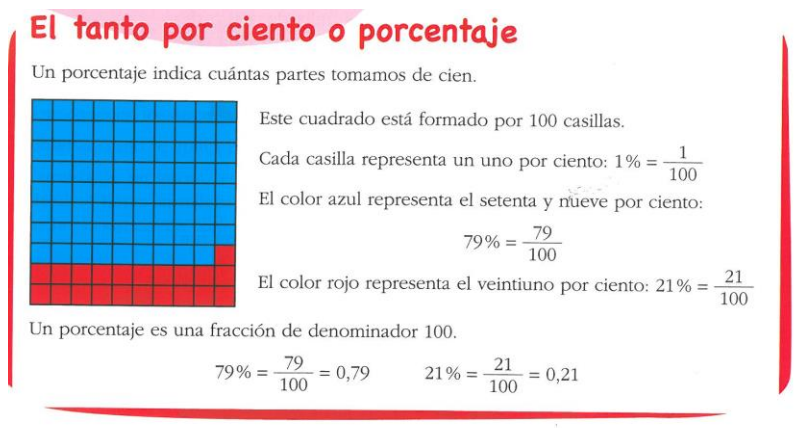
The lesson includes five exercises aimed at:

- Change the representation from the symbolic register to the verbal register (write how the percentages should be read) and within the symbolic register (express the following percentages as a fraction and as decimal numbers).
- Complement to 100 using both symbolic language and natural language:
  - 25% + ... = 100%
  - When buying a sweater there was 30% discount. I paid for the ... percent.
- Calculating the part of quantities of various magnitudes (10% of 150 m).

In this didactical configuration the percentage is presented as a concept from the part-whole model, offering a graphical representation from this point of view that leads to identify the fraction as a pair of numbers that are calculated by means of a double counting. This model can cause difficulties when it comes to introduce percentages higher than 100% (improper fractions).

3.1.2 Didactical configuration 2. Percentage

This configuration (Fig. 2) begins by indicating some cases of use of percentages (%): 'to express how many parts are taken from 100', illustrated with an example of the area model to represent the hundredth part of a square. It concludes with a general definition of percentage as 'fraction of denominator 100', followed by three equivalent representations: percent, fraction, and decimal. The equivalence between fractional and decimal expressions of the numbers involved in the example is assumed to be known.



El tanto por ciento o porcentaje

Un porcentaje indica cuántas partes tomamos de cien.

Este cuadrado está formado por 100 casillas.

Cada casilla representa un uno por ciento: $1\% = \frac{1}{100}$

El color azul representa el setenta y nueve por ciento:

$79\% = \frac{79}{100}$

El color rojo representa el veintiuno por ciento: $21\% = \frac{21}{100}$

Un porcentaje es una fracción de denominador 100.

$79\% = \frac{79}{100} = 0{,}79$ $\quad 21\% = \frac{21}{100} = 0{,}21$

THE PERCENTAGE

A percentage indicates how many parts we take from one hundred.

This box consists of 100 boxes.

Each box represents one percent:

The colour blue represents seventy-nine percent:

The colour red represents twenty-one percent:

A percentage is a fraction with a denominator of 100.

Figure 2. Definition of percentage represented as a fraction.

The objects (previous and emerging) that intervene in this configuration are summarized in Table 3.

Table 3. Intervening objects from didactical configuration 2.

Concepts	Fraction; decimal number Percentage (it is a fraction of denominator equal to 100)
Languages	Natural; symbolic (%, fraction, decimal); graphic (grid)
Procedure	Calculation of the percentage by means of a double counting.
Proposition	The expressions 79/100 and 0.79 are equivalent (equal).

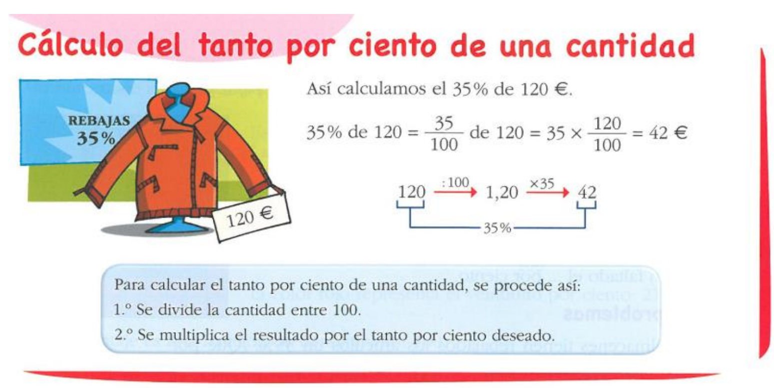
Again, this configuration approach percentages from the part-whole model. There are no arguments and the same difficulty arises: when introducing the percentage by means of a concept-definition and a graphic representation based on the part-whole, a learning obstacle can be anticipated when percentages higher than 100% appear; that is, improper fractions ([16], [17]). On the other hand, the tenths and hundredths of the decimal expression are not related to any of the other registers and linguistic representations; the drawing is simply used to perform a double counting.

Next, there are seven proposed activities, four of which are exercise and three that are applications (problem solving):

- Exercise tasks.
  - Change of representation: 1) from the graphic register to the symbolic register (it expresses as a percentage the colored part of each square) and 2) between the natural and symbolic languages (percentages and decimal fraction) as a table in which it comes given the expression in one of the registers and must be expressed in the other two.
  - Complement up to 100: two tasks identical to those proposed in the didactical configuration 1, in both registers.
- Application tasks. In the three proposed tasks, the complement should be calculated up to 100. For example, it is proposed:
  - In warehouses, items are reduced by 35%. What percentage is paid for each item?

3.1.3 Didactical configuration 3. Calculation of the percent of a quantity

In this configuration (Fig. 3) the procedure to calculate the percentage of a quantity is shown. It is assumed, from the interpretation of the drawing, that in a context of sales and discounts, it is being calculated a 35% discount applied to a coat that initially costs 120 euros.



CALCULATION OF THE PERCENTAGE OF A QUANTITY

This way we calculate 35% of € 120.

(SALES 35%)

To calculate the percentage of a quantity, we proceed as follows:

1º. The amount is divided by 100.

2º. The result is multiplied by the desired percent.

Figure 3. Calculation of the percentage of a quantity.

Table 4 shows the objects related to this configuration.

Table 4. Intervening objects from didactical configuration 3.

Concepts	Multiplication, fraction as operator, fraction, decimal number Percentage
Languages	Natural, symbolic (%, fraction, decimal), diagrammatic
Procedure	Calculation of a discount. It is divided by 100 and then multiplied by the number at the left of the % symbol (which is called percent).

The procedure is explained with an example, but it is not justified considering the previous definition of percentage. Following, nine activities are then proposed to solve situations that involve the calculation of the percentage of a quantity.

- Exercise tasks. Three exercises for the calculation of the part: out of numbers, out of quantities of magnitude and through the percentage as a decimal number. In this last activity, students are expected to calculate certain percentages as the example proposes,

$$15\% \text{ of } 60 = \frac{15}{100} \times 60 = 0.15 \times 60 = 9$$

  and then check the results with the calculator.
- Application tasks. They include three activities on calculation of the part and complement up to 100, two problems that involve the calculation of the part in situation of discount or percentage increase, and another task on calculation of percent.

In addition, two inquiry tasks are included. In one task, students are asked to think of four everyday situations where percentages are used and to write them in their notebook, and then to put them in common with the rest of the classmates. The other task asks students to investigate in relation to VAT (value added tax).

3.1.4 Didactical configuration 4. Review of the lesson

The last part of the unit is exclusively devoted for solving tasks on proportionality and percentages. Out of a total of 22 proposed tasks, a half involves situations about percentages.

- Exercise tasks. Two exercises on calculating percentages of quantities of magnitudes.
- Application tasks. Seven problems are proposed in which the part should be calculated from the whole and the percentage, of which four correspond to situations of discount and two to situations of percentage increase. In addition, a problem is proposed in which students should reflect on the calculation and comparison of percentages and ratios:

  In Carlos's class there are 15 girls and 10 boys. César says: "80% of us are children", Javier says "40% of us are children" and Sara says: "three of every five of us are girls". Who is telling the truth?

3.2 Synthesis of potential epistemic and cognitive conflicts

An instructional process is said to be suitable (adequate) from the epistemic point of view when the implemented meaning matches the intended meaning and this, in turn, to the reference meaning. In our case, the textbook constitutes the local meaning of reference and the sequence of tasks to perform the intended institutional meaning. The mismatches between the institutional meaning of reference and the intended one with the implemented meaning, condition the study process and the the students' learning. These mismatches are called epistemic conflicts. The cognitive suitability of an instructional process expresses the degree to which the intended meanings are in the zone of potential development of the students, as well as the proximity of the achieved personal meanings to the intended or implemented meanings. The disparity between the meanings attributed to an expression by the subject that learns and the intended institutional meanings is understood as a conflict of cognitive type ([5]).

Next, we indicate the epistemic conflicts and potential cognitive conflicts detected in the analysis of the previous didactical configurations.

3.2.1 Epistemic conflicts

In general, the textbook approach to percentages is purely procedural and with no connection with proportionality. It is also not connected to a model that gives meaning, because it is simply stated as a double count in which the second number is always 100. There are no arguments that justify the validity of the procedures or propositions and not all the meanings of the percentage are shown in the lesson analyzed. The proposed activities are mostly exercises, devoting an excessive weight to the calculation of the part from the whole and the percentage and the complement to 100. Only one task has been found (Fig. 4) that mobilizes the reflection on the link of the percentage with the proportionality (without making it explicit, in any case).

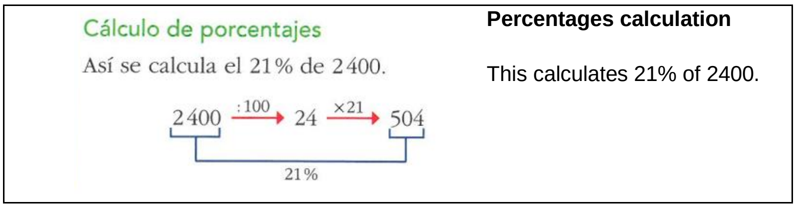


Figure 4. Calculation of percentages.

- In didactical configuration 1, the percentage is presented as a fraction: "indicates how many parts we take from one hundred". Next, the representation is changed by means of percentage to the decimal fraction and to the decimal number without further explanation of the notation used. The percentages are added to complete 100% without further clarification about the meaning of this sum.
- The expression 'percentage of a quantity' appears without connection to the concept of percentage. Now the percentage is used as an operator that is applied to a number. The instructions that appear for this purpose consist of 'divide the amount by 100' and 'multiply the result by the desired percent'.
- In didactical configuration 2, the term 'percent' is used as a synonym of percentage and insists that such a thing is a fraction of denominator 100.
- In the didactical configuration 3, the calculation of the percentage of a quantity is presented, in a context of discounts. The proportionality relationship established between the initial price of a product and the final price established by the discount percentage is not argued at any time.
- On the other hand, in this configuration, the identity:

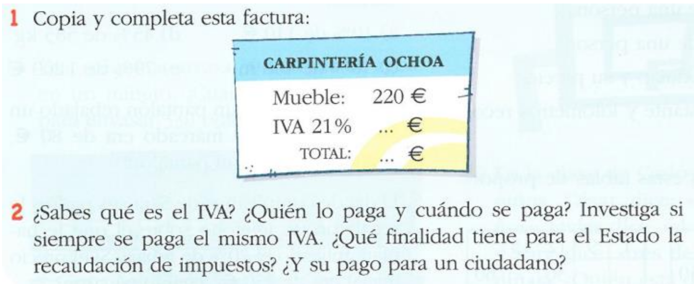
$$\frac{35}{100} \text{ of } 120 = 35 \times \frac{120}{100}$$

  would need some clarification. It should be noted that in the theoretical presentation that had been made in the book of fractions as operator (fraction of a number in unit 6 thereof) the order of operations in the composition was the inverse: first multiplied by the numerator and then it is divided by the denominator.

The part-whole model used has several drawbacks [16]. For example, when it comes to making sense of the percentages above 100, because with double counting the student creates the idea that the number of parts must be less than the total [17].

3.2.2 Potential cognitive conflicts

- Why the topic is called proportionality and percentages?
- Is percent the same as percentage?
- Why 25% of 720 is calculated as $\frac{720}{100} \times 25$ and not as $\frac{25}{100} \times 720$ when 25%=$\frac{\mathbf{25}}{\mathbf{100}}$?
- Why $\frac{35}{100}$ of $120 = 35 \times \frac{120}{100}$?
- In the didactical configuration 3, a task is divided into two parts (Fig. 5): in the first one, the students must complete the invoice of a carpentry, calculating the VAT of 21% on the value of the furniture. In the second part they must answer what is the VAT and who, when, and why that tax is paid. Answering the first part of the activity requires knowing what VAT means.


1 Copia y completa esta factura:

CARPINTERÍA OCHOA	
Mueble:	220 €
IVA 21%	... €
TOTAL:	... €

2 ¿Sabes qué es el IVA? ¿Quién lo paga y cuándo se paga? Investiga si siempre se paga el mismo IVA. ¿Qué finalidad tiene para el Estado la recaudación de impuestos? ¿Y su pago para un ciudadano?

1. Complete the invoice:

CARPENTRY "OCHOA"	
Furniture	€ 220
VAT 21%	€ …
TOTAL	€ …

2. Do you know what is the VAT? Do you know whom, when, and why that tax is paid? Why does the State collect taxes? Why should a citizen pay taxes?

Figure 5. Value Added Tax (VAT).

4 CONCLUSIONS

In this paper we have presented a mathematical textbook analysis technique, applied to a lesson on percentage for 6th grade of primary education, using some theoretical tools from the Onto-Semiotic Approach in mathematics education (OSA).

To assess the adequacy and relevance of the instruction process on percentages planned in the textbook, both the epistemic dimension (relative to institutional meanings) and the cognitive dimension (relative to personal meanings) must be taken into account. In this sense, we summarize below some conclusions of this analysis.

As we mentioned in the previous section, the treatment given to the percentage in the lesson of the textbook is basically algorithmic and devoid of connection with proportionality. Furthermore, as argued by Lundberg and Kilhamn [3], placing proportionality in a separate block, as also occurs in the Swedish national curriculum, does not help teachers make relevant connections between mathematical ideas.

Considering the components and indicators of epistemic suitability, we observe that the connection between percentages and proportionality has not been established in a relevant way, nor have the different meanings of the percentage been shown. We observed that there are no tasks proposed to the students in which the whole or percentage must be determined, nor are percentages higher than 100% considered that require connecting some meanings of the percentage with others. The sample of tasks in which the percentages must be contextualized and applied is not sufficiently representative, being most of the proposed tasks exercise. The weight attributed to the changes of representation (graphic-symbolic and natural language-symbolic) and to the calculation of the part from the whole and the percentage, is excessive, to the detriment of activities that promote proportional reasoning with percentages.

The mathematical language used is appropriate at the level of the students; however, it would be convenient for the textbook to contemplate conversions from symbolic or verbal language to the graphic. The lack of arguments is reflected both in the justifications of the properties and procedures and in the lack of tasks that promote reasoning or argumentation in situations that involve percentages.

From the ecological point of view, it is necessary to propose situations that motivate the interpretation and exchange of information during the resolution of problems that involve percentages in contexts of daily life.

From the cognitive perspective, it is expected that students have the prior knowledge to solve the posed problems. The didactical configuration 1 pursues that the students 'remember' what is a percentage. However, it is not something that has been presented before either in the current course nor in the previous one. The context in which the application tasks are presented is essentially summarized as percentage discount-increase. Only a couple of tasks aimed at the inquiry by the students are proposed, in the didactic configuration 3 (calculation of the percentage of the unit), which foresees to be shared by the group. No tasks are proposed for collaborative work.

Finally, although the authors of the book try to include a variety of problem situations, the results of this study suggest that teachers should take into account theoretical-methodological tools to assess the learning opportunities offered by these tasks and adapt them, if necessary to the needs of each educational level.

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