Curricular Didactic Analysis: an experience with future teachers

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ABSTRACT – Curricular Didactic Analysis: an experience with future teachers. The results of a training experience with future mathematics teachers, focusing on the didactic analysis of curricular materials on probability, are presented. Although there was progress in identifying mathematical objects, difficulties were encountered in recognizing problem-situations, procedures, propositions, and arguments, as well as the meanings of probability. In the expert assessment of the didactic suitability of the standards, shortcomings were observed; however, the participants did not adequately evaluate these shortcomings, particularly in the epistemic and cognitive facets. These limitations could be attributed to a lack of specific training and insufficient time to become familiar with the suitability indicators.
Keywords: **Didactic Analysis. Didactic Suitability. Curricular Material. Probability. Teacher Training.**

RESUMEN – Análisis Didáctico Curricular: una experiencia con futuros profesores. Se presentan resultados de una experiencia formativa con futuros profesores de matemáticas, centrada en el análisis didáctico de materiales curriculares sobre probabilidad. Aunque hubo progreso en la identificación de objetos matemáticos, se encontraron dificultades para reconocer situaciones-problema, procedimientos, proposiciones y argumentos, así como los significados de probabilidad. En la valoración experta de la idoneidad didáctica de la normativa, se observaron deficiencias, pero los participantes no evaluaron adecuadamente dichas carencias, especialmente en las facetas epistémica y cognitiva. Estas limitaciones podrían deberse a la falta de formación específica y al poco tiempo para familiarizarse con los indicadores de idoneidad.
Palabras-clave: **Análisis Didáctico. Idoneidad Didáctica. Material Curricular. Probabilidad. Formación de Profesores.**

Introduction

The training of mathematics teachers emphasizes the importance of developing skills to professionally describe, explain, and assess the teaching and learning processes (Breda; Pino-Fan; Font, 2017; Giacomone et al., 2018; Pino-Fan; Assis; Castro, 2015). Since promoting a critical and reflective approach to the effective use of curriculum materials is essential (Braga; Belver, 2016), Shawer (2017) suggests fostering training in curriculum development, including management of resources such as curriculum programs and teaching guides, among others. Teachers must interpret information in curriculum materials and make adaptations according to contextual needs (Taylor, 2013; Thompson, 2014; Yang; Liu, 2019).

Despite this importance, both pre-service and in-serve teachers often encounter difficulties in critically analysing curriculum materials (Shawer, 2017; Yang; Liu, 2019), making it important to develop specific tools that enable the development of their reflective competence regarding such resources (Remillard; Kim, 2017).

This paper describes a formative experience with future teachers of secondary-level mathematics in Peru, focused on the didactic analysis of curriculum guidelines. We analysed a curriculum program about probability, considering cognitive, social, cultural, and axiological aspects of teaching. Probability is essential in mathematics and has gained importance in recent curricula (CCSSI, 2010; MINEDU, 2016). In particular, the Peruvian education system includes the study of probability from the earliest educational cycles (MINEDU, 2016). However, there is an observed bias towards the classical approach rather than the frequency or subjective meaning, as well as a lack of representativeness of the proposed situations (Cotrado; Burgos; Beltrán-Pellicer, 2022; Vásquez; Alsina, 2015).

For the development of the research, we relied on the Onto-semiotic Approach (OSA) to mathematical knowledge and instruction (Godino; Batanero; Font, 2007). The OSA provides theoretical and methodological tools for the analysis of mathematical activity in curriculum materials and the assessment of the didactic suitability of instructional processes (Godino, 2013; Breda; Font; Pino-Fan, 2018). It also offers a model of knowledge and competencies for mathematics teachers, allowing for the definition of the type of professional knowledge that prospective teachers must acquire in this regard (Godino et al., 2017). These tools are briefly described in the following section.

Theoretical Framework

The research is based on the model of Didactic-Mathematical Knowledge and Competencies (DMKC model) for mathematics teachers, developed within the OSA framework by Godino et al. (2017). This model considers two key competencies for mathematics teachers, *mathematical competence* and the *competence of analysis and didactic* *intervention*, which involves "designing, applying, and evaluating learning sequences developed by oneself and others, using didactic analysis techniques and criteria of quality, in order to establish cycles of planning, implementation, assessment, and propose improvements" (Breda; Pino-fan; Font, 2017, p. 1897). This competence is articulated through five sub-competencies associated with the conceptual and methodological tools of the OSA: analysis of global meanings, onto-semiotic analysis of practices, management of didactic configurations and trajectories, normative analysis, and didactic suitability analysis. In our work, we focus on the sub-competencies of analysis of global meanings, onto-semiotic analysis, and didactic suitability analysis (Godino et al., 2017).

The *analysis of global meanings competency* involves identifying problem-situations and operational, discursive and normative practices implied in their resolution. The teacher must recognize the different meanings of probability: intuitive, laplacian, frequency-based, subjective, and axiomatic (Batanero, 2005; Batanero et al., 2016), how they relate to each other, and how they are addressed in school curricula at different educational levels. The competency of *onto-semiotic analysis of mathematical practices* allows the teacher to identify the diversity of objects and processes involved in mathematical practices necessary for solving problem-situations. This recognition enables the

> [...] anticipation of potential and effective learning conflicts, assessment of students' mathematical competencies, and identification of objects (concepts, propositions, procedures, arguments) that must be remembered and institutionalized at opportune moments in the study processes (Godino et al., 2017, p. 94).

The *competency of analysis of the didactic suitability* of mathematical study processes allows the teacher to assess the appropriateness of the planned or implemented instructional processes and make well-founded decisions for improvement (Godino et al., 2017).

The didactic suitability of an instructional process is defined as the extent to which this process (or a part of it) possesses certain characteristics that allow it to be classified as optimal or suitable for achieving alignment between students' personal meanings attained (learning) and intended or implemented institutional meanings (teaching), taking into account circumstances and available resources (environment). This involves the coherent and systemic articulation of six facets that affect the instructional process: epistemic (specialized mathematical knowledge), cognitive (students' prior knowledge, difficulties, and reasoning), affective (attitudes, beliefs, and emotions of students), interactional (classroom discourse management), mediational (technological and temporal resources), and ecological (curricular adaptation, interdisciplinary and societal relationships) (Godino, 2013; Godino et al., 2007).

For didactic suitability criteria, understood as "[...] corrective norms that establish how a teaching and learning process should be carried out" (Breda; Font, Pino-fan, 2018, p. 264), to be operational, it is necessary to define a set of observable indicators that allow for the assessment of their degree of achievement (Godino, 2013). Furthermore, indicators of didactic suitability should be enriched and tailored, considering the specific content (Breda; Font, Pino-fan, 2018) and the particularity of the instructional process under analysis. For this reason, in Cotrado, Burgos and Beltrán-Pellicer (2022), a systematic review of didactic suitability criteria and indicators is conducted to develop a guide for assessing curriculum materials related to probability.

The aim of our research is to study how the competency of didactic analysis is mobilized and developed in prospective secondary education teachers through formative action, focused on the analysis of the curriculum program related to the topic of probability. Didactic analysis is understood within the OSA as "[...] the systematic study of the factors that condition the teaching and learning processes of a curricular content – or partial aspects of it – with specific theoretical and methodological tools" (Godino et al., 2006, p. 4). It involves, therefore, the analysis of meanings through the identification of practices, onto-semiotic analysis or recognition of the objects involved in these practices, and the assessment of the didactic suitability of the intended or planned instructional process.

Next, we describe the design of the formative action and the process of evaluating the responses provided by the participants.

Methodology

The research approach of this work is interpretive and exploratory in nature, characteristic of design research (Kelly; Lesh; Baek, 2008). It takes place in an authentic classroom context, based on the planning, implementation, and retrospective analysis of an intervention (Godino et al., 2014). Additionally, the methodology of content analysis (Cohen; Lawrence; Morrison, 2011) is employed to examine transcripts of class recordings, as well as participants' response protocols.

Research Context, Participants and Data Collection

The formative experience was conducted at the Faculty of Education of the Universidad Nacional del Altiplano (Peru), involving 14 prospective teachers (PT) from the Mathematics, Physics, Computer Science, and Informatics Program who were taking Descriptive Statistics during the fourth semester of 2021. The intervention comprised theoretical-practical activities and group discussions, utilizing Google Meet and Classroom for synchronous and asynchronous sessions.

Four virtual synchronous sessions were conducted, each lasting two hours. The first two sessions addressed the analysis of global meanings and onto-semiotic analysis, while the last two sessions focused on the analysis of didactic suitability based on curriculum norms.

Session 1: *Development of the competency in the analysis of global meanings and onto-semiotic analysis.* Pragmatic meanings of probability and the network of characteristic mathematical objects were presented. Subsequently, the PTs analysed the curriculum program (CP) (MINEDU, 2023) for the data management and uncertainty block. The CP is divided into 11 units of analysis: NC6 (expected competency level by the end of cycle VI), DG1.1-DG1.5 (first-grade performances), and DG2.1-DG2.5 (second-grade performances). The task for this session involved identifying the mathematical objects and relating them to the meanings of probability emerging in NC6. The session included a group discussion and proposed asynchronous work, which entailed the analysis of DG1.1.

Session 2: *Putting into practice*. The PTs shared and compared their analysis of DG1.1. They continued by individually analysing DG1.2, DG1.3, and DG1.4, and then discussed the results. As an individual asynchronous evaluation task, they were assigned to analyse the units of analysis related to second-grade probability.

Session 3: *Introduction of the didactic suitability analysis tool*. Didactic suitability and its system of components and general empirical indicators were presented as a means of reflection and a rubric for analysing study processes in teaching practice.

Session 4: *Application of the didactic suitability analysis tool*. The PTs applied the suitability indicators to the CP using the guide from Cotrado, Burgos and Beltrán-Pellicer (2022), examining the units of analysis and evaluating whether the suitability indicators were always satisfied, sometimes satisfied, or never satisfied.

Session recordings, trainer's annotations, and written responses to specific tasks were used as data collection instruments.

A priori analysis of the Curricular Programme

The authors conducted didactic analysis of the curriculum program (CP) as a basis for examining the participants' productions. Initially, they analysed the meanings and mathematical objects involved in the units of analysis NC6, DG1.1 to DG1.5, and DG2.1 to DG2.5 of the CP.

In the analysis of NC6, it was determined that the meanings of probability are not clearly identified through the objects associated with each of them (Batanero, 2005), such that situations can be related to both the classical and frequency-based approaches, or even to intuitive aspects (qualitative assessment, predictions). Students are expected to use verbal and symbolic-numeric language registers (fractional, decimal, and integer representations). Specifically, the concepts of random event or situation, probability, sample space, event, certain event, likely event, and impossible event are involved. Procedures were identified such as “enumeration of elementary events,” “relating the value of probability to certain, likely, or impossible events,” and “predicting the occurrence of events.” Propositions such as “the certain event always occurs” and “the probability of an event is associated with a number between 0 and 1” were also found. To justify the propositions in NC6, students are expected to use arguments that support the occurrence of events and the assignment of values between 0 and 1 to certain, likely, and impossible events.

Next, Table 1 exemplifies the expert analysis of the first-grade units of analysis in relation to the mathematical objects observed in the CP.

Table 1 – A priori analysis of the mathematical objects involved in the first-grade units of analysis

Mathematical objects	Analysis units				
	DG1.1	DG1.2	DG1.3	DG1.4	DG1.5
Problem-situations					
Recognizing the conditions that define a random situation	x				
Expressing the value (decimal or percentages) of probability as more or less likely	x	x			
Determining the probability of events using the Laplace rule or calculating their relative frequency	x			x	
Interpreting information from various texts with values or descriptions of random situation			x		
Formulating statements or conclusions about the probability of event occurrences					x
Languages					
Verbal	x	x	x	x	x
Simbolic – numerical	x	x		x	x
Graphical		x	x		
Tabular		x	x		
Concepts					
Random situation	x	x	x	x	
Events, single events	x	x		x	x
More or less probable event	x	x			
Probability	x			x	x
Frequency, relative frequency	x			x	
Decimals, percentages	x			x	
Bar graph, pie chart			x		
Procedures					
Distinguishing the conditions characterising a random situation	x				
Comparison of probability expressed in decimals or percentages	x				
Aplication of Laplace's rule	x			x	
Symbolic or graphical representation					
Reading tables, graphs and texts with random situations			x		
Application of different representations to express the value of probability		x			
Calculation of relative frequency and percentage				x	
Reviewing procedures				x	
Drawing conclusions and correcting errors					x

Propositions					
Laplace’s rule	x			x	
The probability of an event is a calculable value	x	x			
The relative frequency of an event varies between 0 and 1				x	
Arguments					
Justifying the conditions for randomness and which event is more or less likely than another event	x	x			
Making assertions and drawing conclusions					x
Recognising errors in their justifications					x

Source: authors' elaboration.

Regarding the meanings of probability, it was observed that in the DG1.1, DG1.4, DG2.1, and DG2.4 analysis units, terms and expressions related to both the classical and frequency-based approaches are mentioned, such as the use of the Laplace’s rule and the calculation of frequencies or relative frequency. On the other hand, DG1.2, DG1.3, DG1.5, DG2.2, DG2.3, and DG2.5 do not clearly establish a specific approach, which could suggest an orientation towards the classical, frequency-based, or even intuitive approaches. Regarding the frequency-based approach, statistical procedures are included, but not experimentation and simulation. The intuitive approach is identified through qualitative assessments of probability.

After analysing the meanings and mathematical objects, the researchers independently evaluated the suitability of the CP, using criteria and indicators of didactic suitability. This expert analysis will serve as a reference for interpreting the assessments of the PTs. For the degree of fulfillment of an indicator, never, sometimes, and always are assigned 0, 1, and 2 points, respectively. Table 2 illustrates how the different indicators are reflected and to what degree, in relation to the problem-situation components and languages of the epistemic facet, exemplifying the expected analysis by the PTs.

Table 2 – Assessment and identification of indicators according to the components of epistemic suitability

Indicators by components	Fulfilment degree	Analysis units identified in the curricular programme
	Problem-situation	
I1. Proposing the use and formulation of problem-situations that showcase and relate different meanings of probability (intuitive, subjective, frequency-based, and classical).	Sometimes	– Every mathematical activity takes place in the context of solving problems derived from situations, which are conceived as meaningful events occurring in various settings (p. 148) – NC6, DG1.1, DG1.2, DG1.3, DG1.4 y DG1.5, DG2.1, DG2.2, DG2.3, DG2.4 y DG2.5.
I2. Emphasize the formulation of situations where the student generates, experiments with, and simulates problems involving random experiences (problematization).	Never	– The CP does not specify this indicator anywhere in the document.
	Languages	
I3. Promotes the use of different linguistic registers and specific representations of probability such as verbal, symbolic-numeric, tabular and graphical expressions.	Always	– Represents data with graphs and statistical or probabilistic measures: depicts the behaviour of a set of data by selecting statistical tables or graphs (p. 170). – Expresses their understanding of (...) the value of probability using various representations and mathematical language (p. 172).
I4. Appropriate linguistic level for the targeted students.	Always	

Source: Authors' elaboration.

Epistemic Suitability: The CP presents various types of problem-situations without specifying their connection to the meanings of probability, leading to a "sometimes" rating. There is no mention of problematizing situations where students can experiment with or simulate random experiences, resulting in a "never" rating. Concerning languages, the CP promotes the use of suitable registers for the educational level, with an "always" rating. Key concepts, propositions, and procedures are missing in first and second grades, which could introduce biases in learning (Vásquez; Alsina, 2017). Clarifications about relationships between meanings of probability through assimilable mathematical objects are not provided (Batanero, 2005).

Cognitive Suitability: The CP offers general expressions about the progressive treatment of content but does not fully address the diverse meanings of probability (Batanero, 2005). The curriculum focuses on the classical meaning, with little consideration for frequency-based and intuitive meanings, and does not mention common reasoning biases (Lecoutre, 1992).

Affective Suitability: The assessment of alignment with student needs and interests is "always," as the CP clearly states: "[…] that the student analyses data on a topic of interest or study or random situations, allowing them to make decisions, formulate reasonable predictions, and draw conclusions supported by the produced information" (MINEDU, 2016, p. 273). Although the curriculum does not provide specific evidence about emotions, attitudes, and beliefs of students toward random situations, a general record exists for the entire mathematics area that "Emotions, attitudes, and beliefs act as driving forces of learning" (MINEDU, 2016, p. 148); therefore, its alignment is rated as "never."

Interactional Suitability: All indicators in this dimension are only partially observed because the CP does not provide specific interaction guidelines between teacher-student or promotes very generic orientations that encourage communicative interaction among students, as seen in the following expression: "Provide spaces for students for dialogue, debate, discussion, and decision-making, related to their actions or others' actions in various situations" (MINEDU, 2016, p. 25). Autonomy is also promoted through the cross-cutting competency "Manages their learning autonomously" (MINEDU, 2016, p. 29), which applies to all areas.

Mediational Suitability: The use of manipulative and computational resources is not explicitly stated for probability in the curriculum. However, through the expression "[...] use strategies and procedures to collect and process data" (MINEDU, 2016, p. 170), it can be inferred that resources facilitating the calculation of probabilistic measures are implied. Regarding the proper use of classroom space and resources, the CP generally mentions that "The organization of educational spaces, the appropriate and relevant use of educational materials and resources, as well as the teaching role, provide environments and interactions that create a favourable climate for learning" (MINEDU, 2016, p. 54). There are no indications in the CP suggesting the management of suitable schedules or timeframes for addressing probability.

Ecological Suitability: International research and guidelines indicate that meanings of probability should be treated progressively in school curricula (Batanero, 2005; Giacomone et al., 2018; Vásquez; Alsina, 2017). However, this is only partially fulfilled in the CP. No expressions are observed that promote innovation. Socio-professional dimensions, values education, and interdisciplinary connections are presented in a very generic manner. For example, concerning values education, the CP generalizes that

> […] cross-cutting approaches are the observable concretization of the values and attitudes that teachers, students, [...] are expected to demonstrate in the daily dynamics of the educational institution, and which extends to the various personal and social spaces in which they operate (MINEDU, 2016, p. 20).

Analysis of Results and Discussion

In this section, we examine the identification of meanings and mathematical objects by the FPs and their success in evaluating the didactic suitability in the curriculum using the specific indicator guide by Cotrado, Burgos and Beltrán-Pellicer (2022). We also analyse the relationship between the relevance of ontosemiotic analysis and the assessment of didactic suitability compliance in the CP.

Development of the competence in meaning analysis and ontosemiotic analysis

To assess the degree of development achieved in the competence of meaning analysis and ontosemiotic analysis, we begin by examining the difficulties encountered in the initial task (pre-training).

Initial Exploration of Meanings and Mathematical Objects

The initial task aimed to determine the initial conceptions of the PTs regarding mathematical practice and object, using NC6 as the unit of analysis. The PTs correctly identified the presence of verbal and symbolic-numerical language and concepts like probability and sample space, but encountered difficulties with the problem-situations, procedures (only one FP indicated that assigning 0 or 1 as probability could be a procedure), propositions, arguments, as well as recognizing the involved meanings of probability in NC6. Concerning the meanings of probability, four indicated the frequency meaning based on the appearance of statistical tables or predictions. Two PTs related it to the intuitive meaning, while another two considered the classical meaning without justification. Moreover, PT12 suggested that the meaning could be either frequency or classical, depending on the sample space. These limitations might stem from regulatory frameworks prescribing student actions, necessitating the interpretation of descriptions in terms of mathematical objects involved in practices. This finding guided subsequent reflection.

Advancements in the Competence of Meaning Analysis and Ontosemiotic Analysis

In the second session, the PTs analysed the first-grade units (DG1.1 to DG1.4), demonstrating progress in identifying mathematical objects. The quality of the analysis was scored as follows: 0 (no response or all incorrect), 1 (at least one correct object, but less than half), 2 (at least half of the objects correct, but not all), and 3 (all objects correct). Table 3 summarizes the frequency of quality levels exhibited by the PTs in identifying mathematical objects in each first-grade unit of analysis. A slight improvement was observed compared to the first session.

Table 3 – Frequency of the quality of identification of mathematical objects in the analysis conducted by the PTs in the different first-grade units of analysis

Quality of analysis to identify objects	Analysis units			
	DG1.1	DG1.2	DG1.3	DG1.4
Problem-situations				
0 points	12	14	12	11
1 point	1	0	0	0
2 points	1	0	0	0
3 points	0	0	2	3
Languages				
0 points	12	2	1	1
1 point	0	3	1	2
2 points	1	9	11	2
3 points	1	0	1	10
Concepts				
0 points	10	2	2	1
1 point	3	0	8	0
2 points	1	12	4	13
3 points	0	0	0	0
Procedures				
0 points	12	13	10	2
1 point	1	1	4	6
2 points	1	0	0	6
3 points	0	0	0	0
Propositions				
0 points	13	14	14	2
1 point	1	0	0	12
2 points	0	0	0	0
3 points	0	0	0	0

Source: Authors' elaboration.

In the initial analysis of DG1.1, only five out of the 14 PTs actively participated, and only two correctly identified at least one problem-situation (recognizing the conditions defining a random situation) or half of the correct problem-situations. The main difficulties were in identifying procedures, propositions, and arguments. For instance, only two FPs recognized the application of the Laplace's rule as a procedure. The PTs did not recognize that probability is a calculable value and confused the Laplace's rule as a concept rather than a proposition or linked to a procedure.

In the analysis of DG1.2, eleven out of the 14 PTs actively participated, but difficulties in accurately capturing the problems-situations persisted (sometimes descriptions or intent of mathematical practices, e.g., “calculate the probability of situations”), and they confused procedures with propositions (e.g., considering “determine the value of probability” as a proposition). They also had trouble recognizing graphical and tabular language. During the analysis of DG1.3 and DG1.4, there were slight improvements in identifying situations, procedures, and propositions, and justifications started to be indicated as arguments.

After identifying mathematical objects, the preservice teachers were required to link each unit of analysis with underlying probability meanings. Three PTs associated DG1.1 and DG1.4 with the classical meaning, based on the presence of Laplace's rule, and with the frequency meaning due to the inclusion of the relative frequency term. Others attributed an intuitive meaning by using expressions like “more or less likely,” “unlikely,” or “very likely.” Units DG1.2 and DG1.3 were related to classical, frequency, and intuitive meanings, but none of the PTs managed to justify their responses.

In general, the PTs expressed their insecurities and limitations in identifying objects (primarily with propositions and arguments) and meanings. This difficulty may be due, on the one hand, to the lack of training and, on the other hand, to the fact that the curriculum does not explicitly define mathematical entities; rather, it prescribes the actions that the student should perform, requiring the interpretation of emerging practices and objects from these activities. Similarly, in some units of analysis, the standard does not explicitly state expressions that refer to the probability meanings that should be addressed.

Assessment Task: Results of Second-grade Analysis Units by Prospective Teachers

Throughout the workshop implementation, progress was observed in the ontosemiotic analysis capacity of the preservice teachers. The second-grade analysis units (DG2.1 to DG2.5) were used as final assessment instruments following the training sessions. Table 4 presents the frequency of quality levels exhibited by the PTs when identifying mathematical objects in each second-grade analysis unit.

Table 4 – Frequency of the quality of mathematical object identification in the analysis conducted by the PTs in different second-grade analysis units

Quality of analysis to identify objects	Analysis units				
	DG2.1	DG2.2	DG2.3	DG2.4	DG2.5
			Problem-situations		
0 points	1	14	12	5	9
1 point	8	0	2	1	0
2 points	5	0	0	1	0
3 points	0	0	0	7	5
			Languages		
0 points	2	1	1	1	2
1 point	0	2	0	0	1
2 points	0	11	10	2	1
3 points	12	0	3	11	10
			Concepts		
0 points	1	2	2	1	4
1 point	2	2	0	2	4
2 points	11	5	1	9	0
3 points	0	5	11	2	6
			Procedures		
0 points	7	14	6	4	14
1 point	6	0	0	0	0
2 points	1	0	0	3	0
3 points	0	0	8	7	0
			Propositions		
0 points	1	14	14	3	14
1 point	11	0	0	0	0
2 points	2	0	0	0	0
3 points	0	0	0	11	0

Source: Authors' elaboration.

In this activity, the majority of the PTs identified at least one correct problem-situation within the analysis units. However, only a few PTs recognized propositions. In fact, despite reflecting on Laplace's rule as a property, the PTs continued to identify it as a concept. Most procedures were categorized as problem-situations (e.g., calculating probability using Laplace's rule, calculating relative frequencies, comparing event frequencies, reading tables or histogram graphs). Regarding the argument object, only two PTs partially mentioned "justifying results" in DG 2.4 (for example, PT7 considered the presence of arguments in "justifying using the information obtained, and their statistical and probabilistic knowledge"), but none could recognize it adequately.

In all analysis units, the meanings of probability were related to intuitive, classical, and frequency meanings. However, in no case was it justified why they corresponded to these meanings of probability, except for PT6 and PT11, who mentioned that DG2.5 was related to the frequency meaning due to the inclusion of the term relative frequency.

Analysis of didactic suitability carried out by the PTs on the curricular programme

In this section, we present the results of the analysis of the didactic suitability of the curricular regulations carried out by ten PTs, using the analysis tool by Cotrado, Burgos and Beltrán-Pellicer (2022). Tables 5 to 10 summarise the frequencies of the evaluations given by the PTs regarding the fulfilment of suitability indicators in each of the facets. The ones highlighted in italics coincide with the researchers' assessment to verify their correctness.

Regarding epistemic suitability, Table 5 shows a significant disparity with the expert evaluation in aspects related to problem-situations, concepts, procedures, and arguments.

Table 5 – Evaluation by the PTs of the epistemic suitability indicators in the CP

Indicators by components	Assessment			
	Always	Sometimes	Never	No response
Problem-situations				
I1. Proposes the use and formulation of problem-situations that show and relate different meanings of probability (intuitive, subjective, frequentist, and classical).	9	*1*	0	0
I2. Emphasises the formulation of situations where the student generates, experiments, and simulates problems on random experiences (problematization).	6	1	*3*	0
Languages				
I3. Promotes the use of different linguistic registers and specific representations of probability, such as verbal expressions, symbolic-numerical, tabular, and graphical.	*8*	0	0	2
I4. Linguistic level suitable for the target student audience.	*2*	4	1	3
Concepts				
I5. Includes essential concepts: random and deterministic experiment, sample space, event (simple and compound, certain and impossible), favourable and possible cases, frequency, relative frequency, convergence, simulation, experimentation, variability, equiprobability, and probability.	6	*2*	2	0
Propositions				
I6. Proposes the use of propositions and properties such as the probability of the impossible event, certain event and complementary, stability of relative frequencies c, Laplace's rule, and equiprobability.	5	*5*	0	0

Procedures				
I7. Includes procedures for qualitative comparison of probabilities; construction of sample space, application of Laplace's rule, making predictions from observations or data, estimating probabilities, calculating and representing frequencies, using and interpreting diagrams, tables, and graphs, simulating random experiments.	7	*3*	0	1
Arguments				
I8. Recognises the importance of argumentation as a means to demonstrate or justify propositions and solution procedures in which inductive or deductive reasoning may or may not be manifested.	5	*3*	0	2
Relations				
I9. Presents mathematical objects (problems, definitions, etc.) related and connected to each other.	*5*	1	2	2
I10. Recognises and presents the articulation of the various meanings of probability (intuitive, subjective, frequentist, and classical) as an organised whole.	5	0	*3*	2

Source: Authors' elaboration.

Most of the PTs had difficulties identifying correct problem-situations in the ontosemiotic analysis; however, nine of them considered through the fulfilment of I1 the presence of multiple situations in the CP associated with the different meanings of probability. It is possible that the PTs may have misinterpreted this indicator or focused on general expressions. For example, PT3 indicated that the CP promotes the development of competencies to solve data management and uncertainty problems. Although the CP emphasises problem-situations, it lacks precision concerning the meanings of probability that should be integrated into teaching and learning (Batanero, 2005; Beltrán-Pellicer; Godino; Giacomone, 2018).

Problem-situations should include experimentation and simulation (Beltrán-Pellicer; Godino; Giacomone, 2018). The standard provides general clues, which could have confused the PTs. Regarding linguistic records, more than half of the PTs identified the different types of language in the CP but did not know whether they were appropriate for the corresponding level (I4).

Most of the PTs identified at least two or more correct concepts in the CP but assigned inappropriate evaluations to I5, overlooking essential concepts for teaching probability (Batanero, 2005). Similarly, they overlooked basic procedures related to the frequentist meaning (experimentation, estimation, and simulation). Half of the PTs adequately evaluated I6 but did not justify their evaluation, although some identified a correct proposition in the CP. In the regulations, there are implicit arguments, but the use of types of arguments (inductive or deductive) is not proposed. This might have led seven PTs to assign a less relevant evaluation to I8. Regarding the relations component, half of the PTs believed that I10 is always met, despite not having clearly identified the meanings of probability in the CP.

Table 6 summarises the PTs' assessment in the CP concerning the indicators of the cognitive facet. A greater mismatch with the expert evaluation is observed in indicators I11, I14, I15, and I16.

Table 6 – Assessment by the PTs of the cognitive suitability indicators in the CP

Indicators by components	Assessment			
	Always	Sometimes	Never	No response
Prior knowledge				
I11. Suggests progressively working on content according to the meanings of probability.	7	*1*	2	0
I12. Proposes achievable content with a manageable degree of difficulty in the various meanings of probability.	0	*4*	4	2
Cognitive conflicts				
I13. Suggests posing situations to prevent and overcome errors and biases in probabilistic reasoning: representativeness and equiprobability.	3	5	*1*	1
Individual differences				
I14. Promotes access, achievement, and support for all students.	3	*2*	3	2
Assessment				
I15. Provides guidelines on assessment, its procedures, and the application of various techniques and instruments	6	*3*	1	0
I16. Proposes disseminating assessment results to make decisions.	3	2	*4*	1

Source: Authors' elaboration.

Regarding prior knowledge, most of the PTs believed that I11 is always met. They likely did not take into account expressions associated with the progressive treatment of content according to the meanings of probability. Although they justified that the CP includes the different levels of competency development, they did not notice that these expressions only set out linguistic elements and concepts expected at the end of each school cycle.

The CP does not reflect expressions related to cognitive conflicts, but most of the PTs, without justification, assigned partial or total fulfilment evaluations to the associated I13 indicator. Similarly, I14 received inappropriate and unjustified evaluations, possibly because the PTs could not distinguish metacognitive processes according to individual differences. However, PT2 justified the partial fulfilment of the indicator, arguing that according to the CP, "students learn by themselves when they are able to self-regulate their learning process and reflect on their successes, mistakes, progress, and difficulties that arose during the problem-solving process."

Regarding assessment, six PTs believed that I15 is always met, although it is observed partially. For the PTs, it was possibly enough that the CP proposes some generic evaluation instruments. Furthermore, half of the PTs indicated that the CP refers to learning outcomes either partially or always, although this indicator does not appear in the curriculum.

On the affective level (see Table 7), the PTs' assessments in components such as emotions, attitudes, beliefs, and values differ from those of the researchers. The partial or total evaluation of the degree of fulfilment of these indicators was probably due to the identification of imprecise expressions in the CP on the matter, such as features of the same. For instance, PT2 evaluated the indicators I19, I20, I21 with "sometimes", justifying that "Emotions, attitudes, and beliefs act as driving forces of learning" (MINEDU, 2016, p. 148).

Table 7 – Assessment by the PTs of the affective suitability indicators in the CP

Indicators by components	Assessment			
	Always	Sometimes	Never	No response
Interests and needs				
I17. Suggests posing probability situations based on the needs and interests of the students.	*6*	0	3	1
Emotions				
I18. Proposes planning moments where students express emotions in the face of the proposed random situations.	3	2	*4*	1
I19. Suggests posing contextualised random situations and elements that can be motivating.	4	5	*0*	1
I20. Promotes self-esteem, avoiding rejection, phobia, or fear of posing or addressing probability situations or participating in random experiments and simulations.	3	3	*4*	0
Attitudes				
I21. Encourages participation in activities, perseverance, responsibility, etc. to foster a positive attitude towards probability.	5	2	*2*	1
Beliefs				
I22. Guides the teaching-learning process of probability gradually based on the beliefs of the students.	4	3	*0*	3
Values				
I23. Takes into account and highlights the value and utility of chance and probability in the students' daily lives.	5	4	*0*	1

Source: Authors' elaboration.

In Table 8, the discrepancy in the assessments could be due to the fact that the CP does not provide specific guidelines for teacher-student interaction or between students and the development of autonomy (I26, I27, and I28), leading to different assessments than those of the researchers.

Table 8 – Assessment by the PTs of the interactional suitability indicators in the CP

Indicators by components	Assessment			
	Always	Sometimes	Never	No response
Teacher-student interaction				
I24. Suggests appropriate communicative skills for the language of probability.	3	*4*	1	2
I25. Encourages the teacher to use various types of dialogue to guide communicative interaction in the classroom.	6	*2*	2	0
Interaction among students				
I26. Suggests moments that favour dialogue and communication between students.	5	*3*	2	0
I27. Proposes inclusion in the group and avoids exclusion.	5	3	*1*	1
Autonomy				
I28. Suggests autonomous work by students in solving probabilistic situations.	8	*2*	0	0

Source: Authors' elaboration.

Table 9 presents the assessments of the mediational suitability indicators that the PTs assigned when assessing the CP. The general expressions related to material resources, classroom conditions, and time management in the CP may have made it difficult for most of the PTs to assign appropriate assessments of the degree of fulfilment.

Table 9 – Assessment by the PTs of the mediational suitability indicators in the CP

Indicators by components	Assessment			
	Always	Sometimes	Never	No response
Material resources				
I29. Promotes the use of manipulative materials (dice, coins, cards, balls), audiovisuals, and ICT (software, applets, and random devices) to enhance and understand the meanings of probability.	8	*2*	0	0
No. of students, schedule, and classroom conditions				
I30. Suggests employing or prioritising an appropriate schedule to teach probability topics.	3	1	*5*	1
I31. Sets specific guidelines for the proper use of space, equipment, and classroom resources.	4	*3*	2	1
Time				
I32. Proposes managing time in favour of achieving the proposed objectives for teaching probability.	3	3	*3*	1

Source: Authors' elaboration.

In the ecological facet, Table 10 shows that four PTs believe that I33 is "always" met, suggesting that they possibly did not conduct a prior analysis of the curriculum in relation to its correspondence with international research and regulations. In this facet, the indicators where the greatest disparity with the expert assessment is observed are those related to openness to innovation, values education, and intra and interdisciplinary connections. For instance, in the CP, no expressions related to innovation and reflective practice are observed, but most of the PTs assigned "always" or "sometimes" fulfilment assessments to this indicator. Although the CP does not explicitly state the intra and interdisciplinary relationship, references to statistics are observed, leading most of the PTs to assess I37 as "always" met, when actually it is only partially.

Table 10 – Assessment by the PTs of the ecological suitability indicators in the CP

Indicators by components	Assessment			
	Always	Sometimes	Never	No response
Curriculum alignment				
I33. The meanings, their implementation, and assessment of probability align with international curricular guidelines and research.	4	*4*	1	1
Openness to innovation				
I34. Promotes the implementation of innovative activities based on research and reflective practice.	4	2	*2*	2
Socio-professional alignment				
I35. The contemplated probability contents contribute to the socio-professional training of students	4	*4*	2	0
Education in values				
I36. Encourages training in democratic values, inclusiveness, and equal opportunities for making inquiries (critical thinking).	4	*5*	1	0
Intra and Interdisciplinary Connections				
I37. The probability contents relate to other intra and interdisciplinary contents.	5	*1*	3	1

Source: Authors' elaboration.

Relationship between Ontosemiotic Analysis and Didactic Suitability

In this section, we analyse the relationship between the degree of accuracy in identifying mathematical objects in the CP and the success achieved in the analysis of didactic suitability. To observe this relationship, Table 11 collects the frequency of the different didactic suitability assessments made by the PTs. Additionally, it displays the relevance of object identification, based on the quantitative scores 0 (no answer or all incorrect), 1 (at least one correct object, but less than half), 2 (at least half of the objects are correct, but not all) and 3 (all objects are correct).

Table 11 – Frequency of PTs based on object identification scoring and relevance of didactic suitability assessment

Assessment of suitability		Accuracy in the identification of mathematical objects																			
		Problem-situations				Languages				Concepts				Procedures				Propositions			
		0	1	2	3	0	1	2	3	0	1	2	3	0	1	2	3	0	1	2	3
Epistemic																					
Alta	2	0	2	0	0	0	0	0	2	0	0	1	1	0	2	0	0	0	2	0	0
Media	1	0	1	0	0	0	0	0	1	0	0	1	0	0	1	0	0	0	1	0	0
Baja	7	2	5	0	0	0	1	5	1	0	4	2	1	0	7	0	0	2	5	0	0
Cognitive																					
Alta	6	0	6	0	0	0	0	3	3	0	3	2	1	0	6	0	0	0	6	0	0
Media	1	0	1	0	0	0	0	1	0	0	0	1	0	0	1	0	0	1	0	0	0
Baja	3	2	1	0	0	0	1	1	1	0	1	1	1	0	3	0	0	1	2	0	0
Affective																					
Alta	2	0	2	0	0	0	0	1	1	0	0	2	0	0	2	0	0	1	1	0	0
Media	1	0	1	0	0	0	0	0	1	0	0	0	1	0	1	0	0	0	1	0	0
Baja	7	2	5	0	0	0	1	4	2	0	4	2	1	0	7	0	0	1	6	0	0
Interactional																					
Alta	1	0	1	0	0	0	0	1	0	0	1	0	0	0	1	0	0	0	1	0	0
Media	3	0	3	0	0	0	0	2	1	0	2	1	0	0	3	0	0	0	3	0	0
Baja	6	2	4	0	0	0	1	2	3	0	1	3	2	0	6	0	0	2	4	0	0
Mediational																					
Alta	3	0	3	0	0	0	1	2	0	0	0	2	1	0	3	0	0	1	2	0	0
Media	3	0	3	0	0	0	0	2	1	0	2	1	0	0	3	0	0	0	3	0	0
Baja	4	2	2	0	0	0	1	2	1	0	1	1	1	0	4	0	0	1	3	0	0
Ecological																					
Alta	5	0	5	0	0	0	0	3	2	0	2	2	1	0	5	0	0	1	4	0	0
Media	2	1	1	0	0	0	0	1	1	0	0	1	1	0	2	0	0	0	2	0	0
Baja	3	1	2	0	0	0	1	1	1	0	2	1	0	0	3	0	0	1	2	0	0

Source: Authors' elaboration.

In general, all the PTs recognised at least one correct mathematical object in all the analysis units of the CP. However, most were not successful in assessing the suitability of the CP in the epistemic, affective (correctly assessed only by two PTs in each case), interactional (only correctly assessed by one PT), and mediational facets (correctly assessed by three PTs), performing better in the cognitive and ecological facets, correctly assessed by six and five PTs, respectively.

Languages and concepts were the best-referred objects, but only the indicators associated with the language component were appropriately assessed in the epistemic facet. In contrast, the assessment of the indicators related to the concept component was not very relevant both in the epistemic facet (where the PTs did not recognise essential concepts lacking in the CP that are crucial for adequate probability teaching) and in the cognitive one (in the component of prior knowledge, they failed to adequately observe that the CP proposed a partial progressive treatment and manageable difficulty of the content).

Problem situations, procedures, and propositions presented greater identification challenges, and the assessment of related indicators was also not very relevant. Thus, the limited identification of problem-situations influenced the appropriate assessment of indicators related to the affective facet, such as the proposal of problem-situations based on the needs and interests of students and the contextualisation of these with motivating elements.

Conclusions

In this study, we have described the design, implementation, and outcomes of a formative experience with prospective Peruvian mathematics teachers, aimed at promoting the competence of didactic analysis of curricular materials (normative frameworks) on the topic of probability. While these curricular resources are used for planning instructional processes on a specific content, their analysis is a complex task that involves didactic-mathematical knowledge about the content, its teaching, and its learning (Remillard; Kim, 2017).

The distinction of meanings and identification of the mathematical objects involved in these normative frameworks is a challenge for prospective teachers and an essential competence that will allow them to understand and reflect systematically and in detail on the relevance of the teaching and learning processes of probability, considering the educational context. Each meaning of probability involves different systems of practices and, therefore, different challenges and difficulties in its instruction. Properly identifying the types of objects ensures understanding their functionality in these practices. However, as we have seen in our research, participants showed limitations in identifying these objects (especially propositions).

In general, the inadequate identification of mathematical objects in the CP influenced the low relevance of the didactic suitability assessment of the CP. This fact is mainly reflected in the epistemic facet, where only two PTs were successful in their assessment, and it was observed that the PTs overlooked fundamental concepts for adequate probability teaching or decisive procedures related to the frequentist meaning. The difficulty in appropriately assessing indicators from other facets on the affective, interactional, or mediational plane may be motivated both by the challenge in properly interpreting the indicators and by the lack of precision in the programme. Therefore, there is an inferred need for curricular programmes to provide more specific guidelines to teachers and incorporate the results of research in the area of mathematics education so that the standard helps improve the teaching and learning of probability in our case. But also, the need to train prospective teachers to be curriculum developers who reflectively, groundedly, and critically assess the guidelines.

Despite the disparity in assessments between researchers and PTs, the latter were able to identify the main shortcomings and deficiencies in the CP, as had been observed in previous experiences in the context of proportionality (Castillo; Burgos, 2022). Therefore, analysing and evaluating curricular regulations before their use is a good strategy to generate spaces for critical analysis, reflection, and professional development.

The implementation of curricular regulation analysis is a significant contribution to mathematics didactics, as it is not limited to a descriptive aspect and allows analysing the process of contextualising curricula on a specific topic, without neglecting essential aspects such as the affective, interactional, mediational, and ecological. Moreover, it allows diagnosing didactic-mathematical knowledge in relation to the content addressed, making action decisions to correct its deficiencies.

The ontosemiotic analysis and the didactic suitability of the curricular regulation on probability have been challenging for many prospective teachers, possibly due to the lack of specific training and the short time to familiarise themselves with the suitability indicators. It is recommended to specify more those difficult-to-assess indicators and include prior training on facets, components, and didactic suitability indicators in future formative interventions[1].

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